ω-ρ mixing and the $\omega\to\pi\pi\gamma$ decay

Dafne Guetta* and Paul Singer†
Department of Physics, Technion-Israel Institute of Technology, Haifa 32000, Israel

(Received 8 May 2000; published 6 December 2000)

We reexamine the $\omega\to\pi^0\pi^0\gamma$ decay, adding the effect of ω-ρ mixing to the amplitude calculated with the aid of chiral perturbation theory and vector meson dominance. We predict the neutral decay to occur with a width of $\Gamma(\omega\to\pi^0\pi^0\gamma)=(390\pm 96)$ eV and also analyze the effect of the ω-ρ mixing on the $\Gamma(\omega\to\pi^0\pi^0\gamma)/\Gamma(\omega\to\pi^+\pi^-\gamma)$ ratio. Several remarks on the effect of $\omega-\rho$ mixing on certain radiative decays of vector mesons are presented.

DOI: 10.1103/PhysRevD.63.017502 PACS number(s): 12.39.Fe, 12.40.Vv, 13.20.Jf

The radiative decays of mesons have been a subject of continuous interest on both the experimental and theoretical planes since the early 1960s. The main effort has been directed first to the magnetic dipole transitions $V\to P\gamma$, $P\to V\gamma$, where P,V belong to the lowest multiplets of vector (V) and pseudoscalar mesons (P). A large variety of theoretical models has been employed to treat these transitions, such as quark models, bag models, effective Lagrangian approaches, potential models, sum rules and other (Refs. [1–4] provide a comprehensive and complementary list of references). Recently, the interest has focused on such transitions in the sector of the heavy mesons, i.e., $B^*\to B\gamma$, $D^*\to D\gamma$. Here again the models mentioned above have been used, this time in combination with heavy quark effective theories [5–7] (see Ref. [7] for an extensive list of references).

Another class of electromagnetic radiative decays of vector mesons is that in which the final state consists of more than one hadron, such as $\omega\to\pi\pi\gamma$ [8], $\rho\to\pi\pi\gamma$ [9,10], $\phi\to K\bar{K}\gamma$ [11,12], $\phi\to\pi\pi\gamma$ [10,13], and similar decays with one η meson in the final state [14,15]. Although these decays have smaller branching ratios than $V\to P\gamma$ decays, their study offers several attractive new physics features, like the possibility of investigating final state interactions in the hadronic $\pi\pi$ [13,16–18] and $K\bar{K}$ [11,12] channels as well as affording the application of chiral perturbation theory for their calculation [18–20].

The original model [8] for the $\omega\to\pi\pi\gamma$ decay postulated a mechanism involving the dominance of the intermediate vector meson contribution (VMD); i.e., the transition occurs via $\omega\to(\rho)\pi\to\pi\pi\gamma$. Thus, the basic interaction term is the Wess-Zumino anomaly term of the chiral Lagrangian [21] proportional to the Levy-Civita antisymmetric tensor. The interaction term of two vector mesons and one pseudoscalar meson is then given by

$$\mathcal{L}_{V_1V_2\pi}=f_{V_1V_2\pi}\epsilon_{\alpha\beta\gamma\delta}q_1^\alpha\epsilon_1^\beta q_2^\gamma\epsilon_2^\delta, \tag{1}$$

where q_i,ϵ_i are the respective momenta and polarizations (V_i may be a photon). Using this mechanism, the Born amplitude for $\omega\to\pi^0\pi^0\gamma$ decay is given by

$$A^{(B)}(\omega\to\pi^0\pi^0\gamma)=\frac{g_{\omega\rho\pi}g_{\rho\pi\gamma}}{m_\pi^2}\epsilon_{\alpha\beta\gamma\delta}\epsilon_{\nu\delta\tau\psi}p^\alpha\epsilon^{*\beta}(p)p_3^\tau\epsilon^\psi(p_3)\times\left[\frac{P^\gamma P^\nu}{(P^2-m_\rho^2+i\Gamma_\rho m_\rho)}+\frac{Q^\gamma Q^\nu}{(Q^2-m_\rho^2+i\Gamma_\rho m_\rho)}\right], \tag{2}$$

with

$$P=p_2+p_3,\ Q=p_1+p_3, \tag{3}$$

where p_1,p_2 are pion momenta and p_3 and p are the photon and ω momenta. In Eq. (2) we defined new dimensionless g couplings, obtained by dividing the f couplings to m_π.

The decay width of $\omega\to\pi^0\pi^0\gamma$ is proportional to $g_{\omega\rho\pi}^2$ and $g_{\rho\pi\gamma}^2$. Assuming that $\omega\to 3\pi$ proceeds via the same mechanism, $\omega\to(\rho)\pi\to\pi\pi\pi$ [22] and using the experimental input $\Gamma(\omega\to 3\pi)=(7.47\pm 0.14)$ MeV, $\Gamma(\rho^0\to\pi^0\gamma)=(102.5\pm 25.6)$ keV, $\Gamma(\rho\to\pi\pi)=(150.7\pm 1.1)$ MeV [23] one predicts [8,15], using the Born-term amplitude (2),

$$\Gamma^B(\omega\to\pi^0\pi^0\gamma)=\tfrac{1}{2}\Gamma^B(\omega\to\pi^+\pi^-\gamma)=(344\pm 85)\ \text{eV}, \tag{4}$$

where the factor 1/2 is a result of charge conjugation invariance to order α [8] which imposes pion pairs of even angular momentum. In calculating Eq. (4) we used in Eq. (2) a momentum dependent width for the ρ meson [24]

$$\Gamma_\rho(q^2)=\Gamma_\rho\left(\frac{q^2-4m_\pi^2}{m_\rho^2-4m_\pi^2}\right)^{3/2}\frac{m_\rho}{\sqrt{q^2}}. \tag{5}$$

If a constant ρ width is used as frequently done (see, e.g., Ref. [25]), a width of only 306 eV is obtained for $\omega\to\pi^0\pi^0\gamma$ from the Born term. The value in Eq. (4) obtained with presently known coupling constants, updates the different older values in the existing literature.

The branching ratio $\Gamma(\omega\to\pi^0\pi^0\gamma)/\Gamma(\omega\to\pi^+\pi^-\pi^0)$ is independent of $g_{\omega\rho\pi}$ and is a function of $g_{\rho\pi\gamma}$, $g_{\rho\pi\pi}$ only

*Email address: firenze@physics.technion.ac.il
†Email address: phr26ps@physics.technion.ac.il

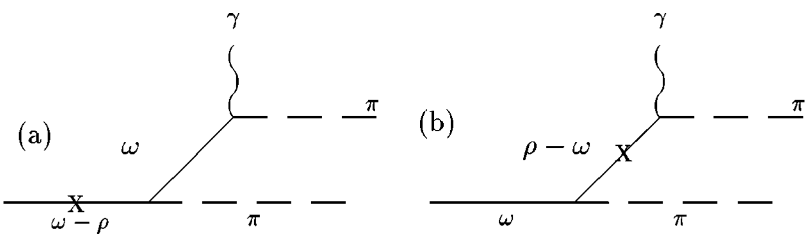

FIG. 1. $\omega\to\pi^0\pi^0\gamma$ decay via mixing: (a) in the wave function, (b) in the propagator.

in this model. Presently there appears to be a discrepancy between the experimental $\rho^0\to\pi^0\gamma$ and $\rho^+\to\pi^+\gamma$ widths. We shall return to this point later. In obtaining Eq. (4) we used the experimental value of $\rho^0\to\pi^0\gamma$ for both the charged and the neutral $\omega\to\pi\pi\gamma$ decays. The quantities mentioned before Eq. (4) give $\Gamma(\omega\to\pi^0\pi^0\gamma)/\Gamma(\omega\to\pi^+\pi^-\pi^0)=(4.6\pm1.2)\times10^{-5}$, using the Born term (4) only. Recently, a new theoretical approach has been advanced for the calculation of $V\to P^0P'^0\gamma$ decays [15] by using the framework of chiral perturbation theory. In Ref. [15] various decays of the $V\to PP'\gamma$ type have been calculated at the one loop level, including both $\pi\pi$ and $K\bar{K}$ intermediate loops. As these authors have shown, the one-loop contributions are finite and to this order no counterterms are required. The calculation [15] has covered the $\phi^0\to\pi\pi\gamma, K\bar{K}\gamma, \pi^0\eta^0\gamma, \rho^0\to\pi^0\pi^0\gamma, \pi^0\eta\gamma$, and $\omega\to\pi^0\pi^0\gamma, \pi^0\eta\gamma$ decays. An improved calculation for the ϕ^0 and ρ^0 decays using unitarized chiral amplitudes [18] leads to comparable numerical results.

Now, in addition to the chiral loop contribution, there is always an additional term in the decay amplitudes given by the intermediate vector meson dominance mechanism [8,14,15]. In the ρ^0 decays, the contribution from pion loops is comparable to that given by VMD term while kaon loops give a minute contribution only. In the ϕ^0 decays, the contribution from kaon loops (pion loops are isospin forbidden) is an order of magnitude larger than the Zweig-forbidden VMD term [19].

On the other hand, a very different and interesting situation arises in the $\omega\to\pi^0\pi^0\gamma$ decays. Here, as a result of isospin invariance, only kaon loops can contribute and the chiral amplitude A_χ obtained [19] is very small; the VMD amplitude $A_{\rm VMD}$ is the dominant feature and leads to a decay width which is two orders of magnitude larger than A_χ gives. Thus, for $\omega\to\pi^0\pi^0\gamma$ we have the remarkable result

$$\begin{aligned} A(\omega\to\pi^0\pi^0\gamma)&=A_\chi(\omega\to\pi^0\pi^0\gamma)\\ &\quad+A_{\rm VMD}(\omega\to\pi^0\pi^0\gamma)\\ &\simeq A_{\rm VMD}(\omega\to\pi^0\pi^0\gamma). \end{aligned} \tag{6}$$

This implies that the decay width for this mode should be essentially accounted for by the amplitude (2). A similar situation is encountered in the $\omega\to\pi^0\eta^0\gamma$ decay [19]; however, in view of the very small branching ratio of $\sim10^{-7}$ expected for it, we shall not discuss further this mode here.

At this point we refer to the experimental situation. For the charged mode there is only an upper limit $\mathrm{Br}(\omega\to\pi^+\pi^-\gamma)<3.6\times10^{-3}$ [23]. On the other hand the neutral mode has been detected by the GAMS Collaboration at HEP [26] and the branching ratio has been measured to be $\mathrm{Br}(\omega\to\pi^0\pi^0\gamma)=(7.2\pm2.6)\times10^{-5}$. Using the well determined ω full width of (8.41 ± 0.09) MeV one arrives at $\Gamma(\omega\to\pi^0\pi^0\gamma)=(0.61\pm0.23)$ KeV. The central value of this result is nearly twice the VMD result of Eq. (4). This lead us to reexamine the mechanism of this decay, especially in light of its unique position described above, which requires $A(\omega\to\pi^0\pi^0\gamma)\simeq A_{\rm VMD}(\omega\to\pi^0\pi^0\gamma)$.

Within the theoretical framework just described, based on chiral perturbation theory and vector meson dominance, there is one feature which has been neglected so far. This is the possibility of ω-ρ mixing [27] which, for example, is responsible for the isospin violating $\omega\to\pi^+\pi^-$ decay, occurring with a branching ratio of [23] $\mathrm{Br}(\omega\to\pi^+\pi^-)=(2.21\pm0.30)\%$.

We proceed now to investigate whether the ω-ρ mixing could possibly account for the existing discrepancy between the central values of the theoretical (4) and experimental results.

The mixing between the isospin states $\rho^{(I=1)}$, $\omega^{(I=0)}$ may be described by adding to the effective Lagrangian a term $\mathcal{L}=\mathcal{M}^2_{\rho\omega}\omega_\mu\rho^\mu$, which leads to the physical states

$$\rho=\rho^{(I=1)}+\epsilon\omega^{(I=0)},\quad \omega=\omega^{(I=0)}-\epsilon\rho^{(I=1)}, \tag{7}$$

where [27]

$$\epsilon=\frac{\mathcal{M}^2_{\rho\omega}}{m^2_\omega-m^2_\rho+im_\rho\Gamma_\rho-im_\omega\Gamma_\omega}. \tag{8}$$

Using the experimental values for m_ρ, m_ω, Γ_ρ, Γ_ω [23] and $\mathcal{M}^2_{\rho\omega}=-(3.8\pm0.4)\times10^3\ \mathrm{MeV}^2$ as determined from fits to $e^+e^-\to\pi^+\pi^-$ [27], one obtains

$$\epsilon=-0.006+i0.036. \tag{9}$$

The effect of the ω-ρ mixing is to add to the Born diagram the two diagrams of Fig. 1, expressing the mixing of ρ into the ω wave function (7) as well as the modification arising from mixing in the ρ propagator [25]. As a result, the full amplitude for $\omega\to\pi^0\pi^0\gamma$ decay is given by

$$\begin{aligned} A^{(\omega-\rho)}(\omega\to\pi^0\pi^0\gamma)&=\tilde{A}(\omega\to\pi^0\pi^0\gamma)\\ &\quad+\epsilon A^{(B)}(\rho^0\to\pi^0\pi^0\gamma), \end{aligned} \tag{10}$$

where $\tilde{A}$ has the form of Eq. (2) with the ρ propagator replaced by

$$\frac{1}{P^2-m_\rho^2+im_\rho\Gamma_\rho}\to\frac{1}{P^2-m_\rho^2+im_\rho\Gamma_\rho}+\frac{f_{\omega\pi\gamma}}{f_{\rho\pi\gamma}}\frac{\mathcal{M}_{\rho\omega}^2}{(P^2-m_\rho^2+im_\rho\Gamma_\rho)(P^2-m_\omega^2+im_\omega\Gamma_\omega)}. \tag{11}$$

In the second term in Eq. (10) $A^{(B)}(\rho^0\to\pi^0\pi^0\gamma)$, has the same expression as Eq. (2), except that ω,ρ are interchanged everywhere. Calculating now the decay width from Eq. (10) we find

$$\Gamma^{(\omega-\rho)}(\omega\to\pi^0\pi^0\gamma)=(363\pm90)\ \text{eV}. \tag{12}$$

Thus, ω-ρ mixing increases the $\omega\to\pi^0\pi^0\gamma$ width by 5% only, even less than the 12% increase provided by the q^2 dependence of Γ_ρ, as discussed after Eq. (4). The newly calculated value is still about half the experimental one [23,26] of $\Gamma^{(\exp)}(\omega\to\pi^0\pi^0\gamma)=(610\pm230)$ eV. We have checked the effect of using the experimental value of $\rho^+\to\pi^+\gamma$ instead of $\rho^0\to\pi^0\gamma$ and the effect of the relatively slight changes in the value of $\mathcal{M}_{\rho\omega}^2$, as given in the literature [27], and we found that the result given in Eq. (12) is practically not changed. It should also be mentioned at this point that the uncertainty in Eq. (12) is mostly due to the uncertainty in $\Gamma(\rho^0\to\pi^0\gamma)$.

Since ω-ρ mixing turns out to be a small effect here also, we combine now all the improvements on the simple Born term of [8], i.e., ω-ρ mixing, q^2 dependence of Γ_ρ and the inclusion of the A_χ term as given in Eq. (4) of Ref. [19]. Using the amplitude which contains all these effects we predict

$$\Gamma^{th}(\omega\to\pi^0\pi^0\gamma)=(390\pm96)\ \text{eV}. \tag{13}$$

Before concluding, we present a few remarks related to the effects of ω-ρ mixing on various radiative decays of vector mesons.

The $\omega-\rho$ mixing expressed in Fig. 1 affects the neutral mode $\omega\to\pi^0\pi^0\gamma$, hence the 1/2 ratio of Eq. (4) between the neutral and the charged modes will be affected as well, becoming slightly larger. This is understandable, since the 1/2 factor holds to the first order in α, while the amplitude (10) contains terms of order e^3. At this point, it is important to refer to the photon spectrum of the $\omega\to\pi\pi\gamma$ decay which, as shown in Refs. [8,15] peaks very strongly around 325 MeV. This is the typical spectrum of the direct transition, as driven by the Born term of Eq. (2). There is, however, an additional effect to this mode which was pointed out by Fajfer and Oakes [14] and is caused by the bremsstrahlung radiation $\omega\to\rho\to\pi^+\pi^-\gamma$ [9], emitted following an ω-ρ transition. In this case, contrary to the situation discussed in this paper, only the $\omega\to\pi^+\pi^-\gamma$ decay will be affected. The 1/2 factor will change again, this time in the opposite direction, becoming as small as $\sim 1/5$ [14]. This effect holds however, mostly for the lower part of the photonic spectrum, being due to the bremsstrahlung radiation of ρ^0 and practically dies out beyond $E_\gamma\sim 250$ MeV. Hence both effects can be experimentally tested if enough events are collected to separate the spectra at the photon energy of about 300 MeV.

A second remark refers to statements made in the literature which attribute to ω-ρ mixing or other isospin/$SU(3)$ breaking effects the inducement of a very large deviation from 1 of the ratio $R=\Gamma(\rho^0\to\pi^0\gamma)/\Gamma(\rho^+\to\pi^+\gamma)$, predicting as much as $R=2.4$ [28] or 1.7 ± 0.1 [29]. Unfortunately, these are based on an erroneous formulation of vector meson mixing. Reference [27] presents in detail the correct treatment for this problem. Using Eqs. (7) and (8) to calculate the effect of ω-ρ mixing on R, we expect $R=\Gamma(\rho^0\to\pi^0\gamma)/\Gamma(\rho^+\to\pi^+\gamma)=1.03$ which is consistent with the experimental figure [23] of $R^{\exp}=1.51\pm0.54$.

We also wish to remark that we did not discuss here effects of final state interactions. These are not expected to change the prediction for the rate [16,18], although may affect somewhat the decay spectrum. In any case, if the discrepancy we discussed survives after more accurate experiments, this point should be reexamined as well.

We summarize by stressing the importance of a good measurement of the $\omega\to\pi^0\pi^0\gamma,\omega\to\pi^+\pi^-\gamma$ decay modes. The particular features of this decay arising from the application of chiral perturbation and vector meson dominance [8,15,19] were supplemented here by the inclusion of ω-ρ mixing. Taking all these contributions into account, we predict $\Gamma(\omega\to\pi^0\pi^0\gamma)=(390\pm96)$ eV, which is smaller though barely consistent with the existing experimental value [26] of $\Gamma^{(\exp)}=610\pm230$ eV. The measurements of both channels and of their spectra, will afford also the detection of the α^3 effect which we described and will allow to determine whether a serious discrepancy between theory and experiment occurs in this case.

The research of P.S. has been supported in part by the Fund for Promotion of Research at the Technion.

[1] P. O'Donnell, Rev. Mod. Phys. **53**, 673 (1981).
[2] P. Singer and G.A. Miller, Phys. Rev. D **33**, 141 (1986).
[3] S.I. Dolinsky *et al.*, Phys. Rep. **202**, 99 (1991).
[4] N. Barik and P.C. Dash, Phys. Rev. D **49**, 299 (1994); M. Benayoun *et al.*, *ibid.* **59**, 114027 (1999).
[5] H.-Y. Cheng *et al.*, Phys. Rev. D **47**, 1030 (1993).
[6] P. Cho and H. Georgi, Phys. Lett. B **296**, 408 (1992); J.F. Amundson *et al.*, *ibid.* **296**, 415 (1992).
[7] P. Singer, Acta Phys. Pol. B **30**, 3849 (1999).
[8] P. Singer, Phys. Rev. **128**, 2789 (1962).
[9] P. Singer, Phys. Rev. **130**, 2441 (1963); **161**, 1964(E) (1967).
[10] S.M. Renard, Nuovo Cimento A **62**, 475 (1969).
[11] S. Nussinov and T.N. Truong, Phys. Rev. Lett. **63**, 2002 (1989); J. Lucio and J. Pestiau, Phys. Rev. D **42**, 3253 (1990);

43, 2447(E) (1991).
[12] F.E. Close, N. Isgur, and S. Kumano, Nucl. Phys. **B389**, 513 (1993); J.A. Oller, Phys. Lett. B **426**, 7 (1998).
[13] N.N. Achasov and V.N. Ivanchenko, Nucl. Phys. **B315**, 465 (1989); N.N. Achasov, V.V. Gubin, and E.P. Solodov, Phys. Rev. D **55**, 2672 (1997); G. Colangelo and P.J. Franzini, Phys. Lett. B **289**, 189 (1992); A. Bramon, G. Colangelo, and M. Greco, *ibid.* **287**, 263 (1992).
[14] S. Fajfer and R.J. Oakes, Phys. Rev. D **42**, 2392 (1990).
[15] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett. B **283**, 416 (1992).
[16] N. Levy and P. Singer, Phys. Rev. D **3**, 2134 (1971).
[17] N.N. Achasov and V.V. Gubin, Phys. Rev. D **57**, 1987 (1998).
[18] E. Marco, S. Hirenzaki, E. Oset, and H. Toki, Phys. Lett. B **470**, 20 (1999).
[19] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett. B **289**, 97 (1992).
[20] K. Huber and H. Neufeld, Phys. Lett. B **357**, 221 (1995).
[21] J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971); E. Witten, Nucl. Phys. **B223**, 422 (1983).
[22] M. Gell-Mann, D. Sharp, and W.G. Wagner, Phys. Rev. Lett. **8**, 261 (1962).
[23] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[24] H.B. O'Connell, B.C. Pearce, A.W. Thomas, and A.G. Williams, Phys. Lett. B **354**, 14 (1995); G.J. Gounaris and J.J. Sakurai, Phys. Rev. Lett. **21**, 244 (1968).
[25] P. Langacker, Phys. Rev. D **20**, 2983 (1979).
[26] GAMS Collaboration, D. Alde *et al.*, Phys. Lett. B **340**, 122 (1994).
[27] For a recent review see H.B. O'Connell, B.C. Pearce, A.W. Thomas, and A.G. Williams, Prog. Part. Nucl. Phys. **39**, 201 (1997).
[28] J.L. Diaz-Cruz, G. Lopez Castro, and J.H. Munoz, Phys. Rev. D **54**, 2388 (1996).
[29] M. Hashimoto, Phys. Lett. B **381**, 465 (1996).